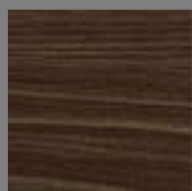
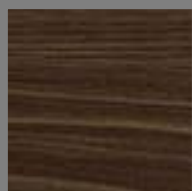
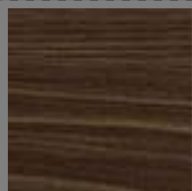


Geometric Pools

Lesson 1

A Lesson in Algebraic Applications of Geometry Incorporating Technology

In this lesson, you will be designing square swimming pools that are surrounded by wooden decks. You will be using tiles to create swimming pools of different widths, recording the size of each pool and deck into tables, and developing rules for finding the area of each pool/deck. At the end of the lesson, you will answer some written reflection questions about what you've learned.



Swimming Pool
Tile



Deck Tile



Scale Factor:
 $\frac{1}{2}$ inch = 1 foot

Part A: Designing Pools by Continuing Patterns

- The first two designs have been created for you. Pool Design 1 has a width of 1 foot, and Design 2 has a width of 2 feet. Continue this pattern using the sample tiles (above) to create Designs 3, 4, and 5.
 - Create the pool by copying the sample pool tile, pasting the number of tiles needed for the design, and dragging the tiles into the appropriate space below.
 - Follow the same procedures using the deck tiles to create a deck that surrounds each pool.
- After creating each design, you will need to “group” the tiles in order to lock them together. To do this, press and hold “CTRL” and select each tile in the design. Then, right click with your mouse, select “GROUPING,” and click on “GROUP” from the dropdown menu.

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Part B: Determining the Area of each Pool


1. Use the sketches you created in Part A to determine the pool width and pool area for Designs 1-5. Record this information in "Table 1: Pool Area."
2. Look for any patterns in the table. Without sketching the designs, continue filling in the table for Designs 6-10.
-  3. Think about the relationship between the width of the pool and its area... Use this relationship to find and record the area of a pool with a width of 30 feet.
4. Using this same relation, write an expression for the area of a pool with width " w ."

Table 1: Pool Area

Design #	Width of Pool (ft)	Area of Pool (ft ²)
1	1	1
2	2	
3		
4		
5	5	
6		
7		
8		
9		
10		
<hr/>		
30		
w		



NOTE: We use a zigzag line to represent a break in the recursive (vertical) pattern. Keep this in mind when completing your table!

Part C: Determining the Size of each Deck

1. Create a table with 4 columns to organize the following information:
 - a. Design Number
 - b. Pool Width
 - c. Perimeter of the Pool
 - d. Size of Deck (number of deck tiles needed to surround the pool)
2. Include a title for your table and correctly label each column, including units of measurement where necessary. Improve the visual quality of your table by customizing the borders, shading and/or text. Be creative!
3. Fill in the table for Designs 1-5, using your sketches from Part A.



NOTE: Finding the perimeter of the pool is not the same as finding the size of the deck! The perimeter of Design 1 is 4 feet, while the deck size is 8 square feet! For Design 2, the perimeter is 8 feet while the deck is 12 square feet!



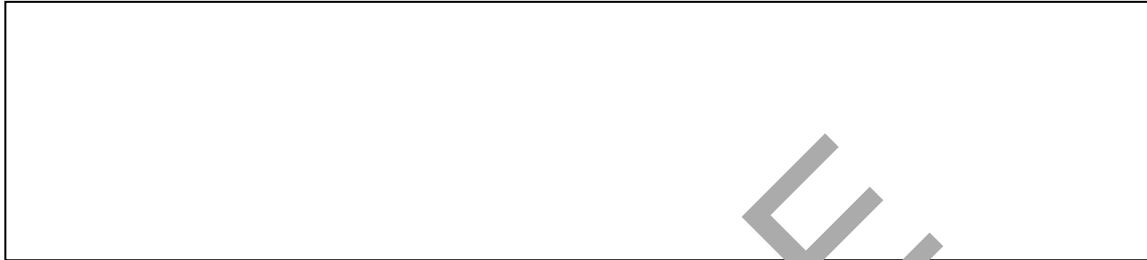
4. Look for any patterns in the table. Without sketching the designs, continue filling in the table for Designs 6-10.
5. Think about the relationship between the width of the pool, the perimeter of the pool and the size of the deck... Use this relationship to find and record the size of the deck for a pool with a width of 30 feet.
6. Using this same relation, write an expression to represent the size of the deck for a pool with width " w ."

SAMPLE

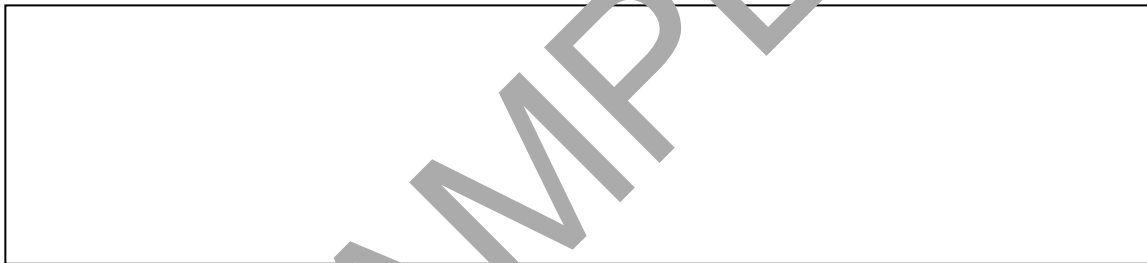
Part D: Written Reflections to Apply, Connect, and Extend Knowledge

Answer each of the following questions by typing your response into the text box below the question. Please choose a new font (type, style, and/or color) for your responses. All answers must be written in complete and grammatically correct sentences.

1. Explain how you constructed the sketches for Pool Designs 3-5.



2. Refer to "Table 1: Pool Area" on page 3. Describe any patterns you notice.



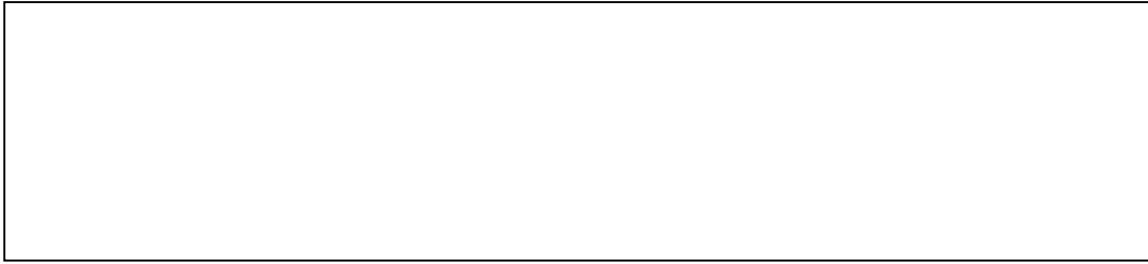
3. Explain how you can find the area of a pool when you are given either the design number or the width of the pool. Give an example to support your response.



4. What is the width of a pool that has an area of 400 square feet? Explain (in words) how you found your answer.



5. Refer to the table you created (on page 4) for finding the size of the deck. Describe any patterns you notice.



6. What is the relationship between the number of tiles needed to create the deck and the perimeter of the pool?



7. Explain how you can find the size of the deck when you are given either the design number or the width of the pool. Give an example to support your response.



8. What is the area of the pool that has a deck of 52 square feet? Explain how you found your answer.



For questions 9-11, assume the cost of building the pool and deck are as follows:

Pool: \$150 per square foot

Deck: \$75 per square foot

9. What is the total cost of Pool Design #8? Show your work or explain how you found your answer.



10. What is the total cost of Pool Design #20? Show your work or explain how you found your answer.



BONUS: If you have a budget of \$30,000, what is the width of the largest pool you can build? Explain how you found your answer.



Scoring Rubric

	Objective	Assessment	Points Possible
Part A: Continuing Patterns	Continue Geometric Patterns	Successfully create pool designs (numbers 3, 4, and 5)	20
	Group Objects	Group the tiles from pool Designs 3, 4 and 5	10
Part B: Pool Area	Enter Data into Table	Complete the "Pool Area" table	5
	Use Recursive Patterns	Enter the correct data into the "Pool Area" table (Designs 1-10)	10
	Use Explicit Patterns	Enter the correct data into the "Pool Area" table (width of 30 feet)	10
	Write an Explicit Function Rule	Enter the correct rule for finding the area of Pool "n" into the "Pool Area" table	15
Part C: Deck Size	Create a Table	Create a new table for finding "Deck Size", include proper headings and units of measurement	20
	Format a Table	Use borders and/or shading to improve the visual quality of the "Deck Size" table, change cell alignment, create a title	10
	Enter data into Table	Complete the "Deck Size" table	5
	Use Recursive Patterns	Enter the correct data into the "Deck Size" table (Designs 1-10)	10
	Use Explicit Patterns	Enter the correct data into the "Deck Size" table (width of 30 feet)	10
	Write an Explicit Function Rule	Enter the correct rule used to find the size of the deck for a pool with width "w"	15
Part D: Reflection	Word Processing & Text Formatting	Enter answer to reflection questions in the given text boxes, change the font (type, size, and/or color), use proper spelling/grammar	10
	Applications, Connections, and Extensions	Answer reflection questions (in complete sentences). Provide justification for your responses. (NOTE: Some questions may have more than one correct response.)	50
TOTAL			200

Geometric Pools

Lesson 1

A Lesson in Algebraic Applications of Geometry Incorporating Technology

In this lesson, students will be designing square swimming pools that are surrounded by wooden decks. They will be using tiles to create swimming pools of different widths, recording the size of each pool and deck into tables, and developing both recursive and explicit rules for finding the area of each pool/deck. At the end of the lesson, students will answer reflection questions that encourage verbal and written communication in mathematics while solidifying their understanding of the connections between mathematical concepts.

Lesson Overview

This is an introductory lesson in a unit on patterns, algebraic thinking, and connections to other mathematical concepts. The overall objective of this lesson is to familiarize students with pattern recognition and function rules. Future lessons in this unit will extend into more complex functions and include graphical representations of linear and nonlinear functions. Questions are open-ended and based in a real-world context in order to encourage independent thinking and creativity in problem-solving. Although this unit can be completed by students with limited teacher input, collaboration is encouraged, as it will expose students to various methods of problem solving. The role of technology in this unit is that of a “mindtool,” where students use various tools to extend patterns, organize and evaluate information, collaborate with their peers and exchange feedback, and make/test conjectures. Although this project could be completed without the use of computers, providing students with technology allows them to create, extend, and generalize complex patterns more efficiently while gaining practical experience using technological tools that they must acquire to become productive members of today’s society. This lesson is designed for students in middle grades (5-8).

Prerequisite Skills

This lesson assumes students have a basic knowledge of word processing programs and the use of tables to organize mathematical data. Students must also be familiar with copying and pasting objects in a word processing program. Students must also have a basic knowledge of typing and text editing, specifically within text boxes.

Students should have had previous exposure to the concept of variables as they are used to represent unknown or changing values. Students with limited exposure to this concept may require assistance in defining an explicit function rule in algebraic terms. Students should have a conceptual understanding of perimeter and area of two-dimensional figures.

Content Standards (NCTM)

- **Write, interpret, and use mathematical expressions and equations. (Grade 6 Focal Point)**
- Develop an initial conceptual understanding of different uses of variables
- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations
- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships
- Describe, extend, and make generalizations about geometric and numeric patterns
- Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions
- Build and draw geometric objects
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life

Process Standards (NCTM)

- Problem-Solving
- Reasoning & Proof
- Communication
- Connections
- Representation

Materials

- Word processing program (MS Word 2007 preferred)
- Calculator
- 1-inch square tiles in 2 colors (optional)

Procedures

Students will follow the step-by-step student guide and record their responses in the appropriate spaces on the document. After completing the unit, it is recommended that students save the document and submit their final work via email.

If this lesson is used in the traditional classroom setting, it would be beneficial for students to print a copy of the document for in-class collaboration. Encourage students to share their responses and reasoning with a small group and with the whole class. Student responses should be shared in order of increasing complexity.

Modifications

Suggested modifications for supporting students with learning challenges:

- Allow students to complete the assignment with a partner or in a small group. Encourage students to share their responses verbally before recording them on the document.
- Have students use manipulatives (such as square tiles) to construct the designs before creating them on the computer. Have extra materials available for students to check their calculations for Designs 6-10.
- Encourage students to visualize the pool and the deck as separate components.

Suggested modifications for enriching advanced students:

- Encourage students to connect their mathematical expressions to the geometric pattern. Encourage students to find multiple ways of justifying their responses.
- Encourage students to come up with and exchange other application questions.
- Have students create a new “design” for the pool and complete parts A-D for this design.

Additional Resources for Teachers

- <http://www.nctm.org/teachingtips.aspx>
- <http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>

Assessment/Evaluation

Use the following rubric to assign a score to your students. This rubric is based on a 70% emphasis on mathematical processes and a 30% emphasis on the technology components of the project.

	Objective	Assessment	Points Possible
Part A: Continuing Patterns	Continue Geometric Patterns	Successfully create pool designs (numbers 3, 4, and 5)	20
	Group Objects	Group the tiles from pool Designs 3, 4 and 5	10
Part B: Pool Area	Enter Data into Table	Complete the "Pool Area" table	5
	Use Recursive Patterns	Enter the correct data into the "Pool Area" table (Designs 1-10)	10
	Use Explicit Patterns	Enter the correct data into the "Pool Area" table (width of 30 feet)	10
	Write an Explicit Function Rule	Enter the correct rule for finding the area of Pool "w" into the "Pool Area" table	15
Part C: Deck Size	Create a Table	Create a new table for finding "Deck Size", include proper headings and units of measurement	20
	Format a Table	Use borders and/or shading to improve the visual quality of the "Deck Size" table, change cell alignment, create a title	10
	Enter data into Table	Complete the "Deck Size" table	5
	Use Recursive Patterns	Enter the correct data into the "Deck Size" table (Designs 1-10)	10
	Use Explicit Patterns	Enter the correct data into the "Deck Size" table (width of 30 feet)	10
	Write an Explicit Function Rule	Enter the correct rule used to find the size of the deck for a pool with width "w"	15
Part D: Reflection	Word Processing & Text Formatting	Enter answer to reflection questions in the given text boxes, change the font (type, size, and/or color), use proper spelling/grammar	10
	Applications, Connections, and Extensions	Answer reflection questions (in complete sentences). Provide justification for your responses. (NOTE: Some questions may have more than one correct response.)	50
TOTAL			200

Swimming Pool
Tile



Deck Tile



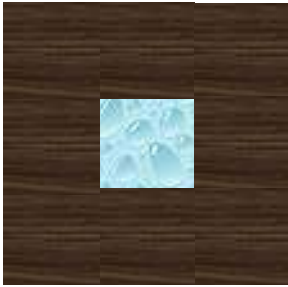
Scale Factor:
 $\frac{1}{2}$ inch = 1 foot

Part A: Designing Pools by Continuing Patterns

- The first two designs have been created for you. Pool Design 1 has a width of 1 foot, and Design 2 has a width of 2 feet. Continue this pattern using the sample tiles (above) to create Designs 3, 4, and 5.
 - Create the pool by copying the sample pool tile, pasting the number of tiles needed for the design, and dragging the tiles into the appropriate space below.
 - Follow the same procedures using the deck tiles to create a deck that surrounds each pool.
- After creating each design, you will need to “group” the tiles in order to lock them together. To do this, press and hold “CTRL” and select each tile in the design. Then, right click with your mouse, select “GROUPING,” then click on “GROUP” from the dropdown menu.

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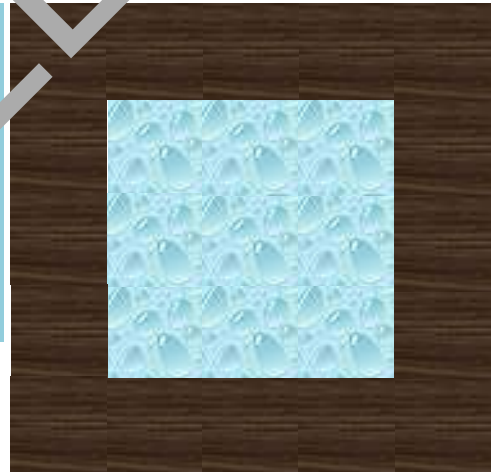
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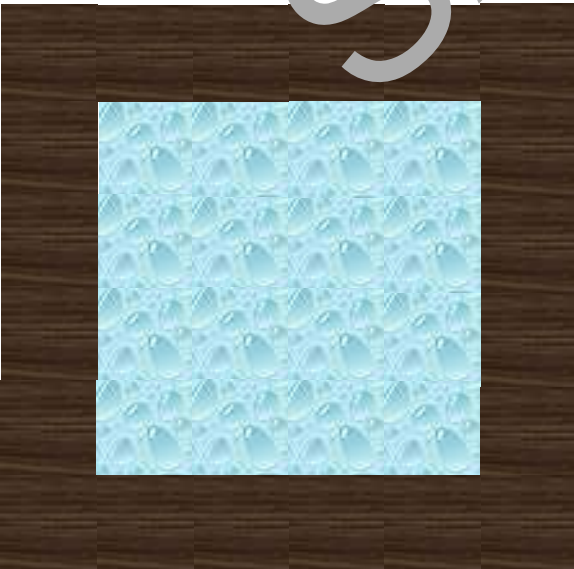
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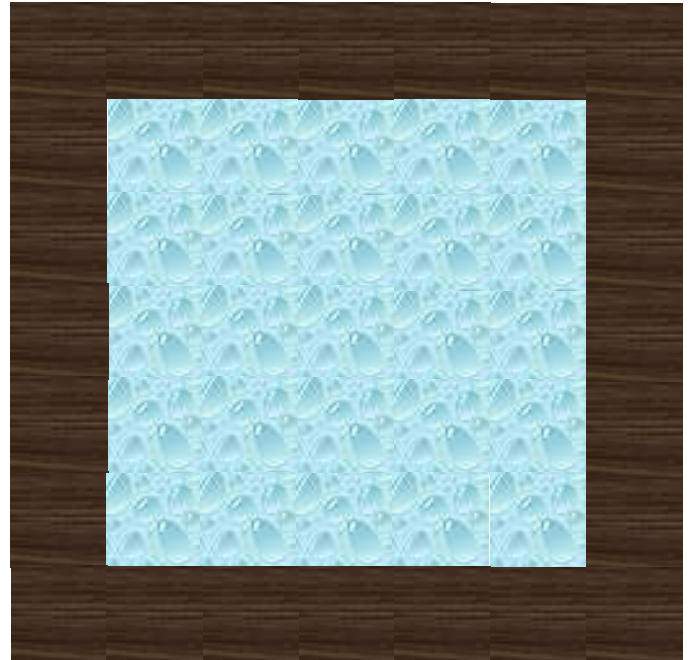
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Part B: Determining the Area of each Pool


1. Use the sketches you created in Part A to determine the pool width and pool area for Designs 1-5. Record this information in "Table 1: Pool Area."
2. Look for any patterns in the table. Without sketching the designs, continue filling in the table for Designs 6-10.
-  3. Think about the relationship between the width of the pool and it's area... Use this relationship to find and record the area of a pool with a width of 30 feet.
4. Using this same relation, write an expression for the area of a pool with width "w."

Table 1: Pool Area

Design #	Width of Pool (ft)	Area of Pool (ft ²)
1	1	1
2	2	4
3	3	9
4	4	16
5	5	25
6	6	36
7	7	49
8	8	64
9	9	81
10	10	100
<hr/>		
30	30	900
w	w	$w \cdot w$ or w^2



NOTE: We use a zigzag line to represent a break in the recursive (vertical) pattern. Keep this in mind when completing your table!

Part C: Determining the Size of each Deck

- Create a table with 4 columns to organize the following information:
 - Design Number
 - Pool Width
 - Perimeter of the Pool
 - Size of Deck (number of deck tiles needed to surround the pool)
- Include a title for your table and correctly label each column, including units of measurement where necessary. Improve the visual quality of your table by customizing the borders, shading and/or text. Be creative!
- Fill in the table for Designs 1-5, using your sketches from Part A.

NOTE: Finding the perimeter of the pool is not the same as finding the size of the deck! The perimeter of Design 1 is 4 feet, while the deck size is 8 square feet! For Design 2, the perimeter is 8 feet while the deck is 12 square feet!

- Look for any patterns in the table. Without sketching the designs, continue filling in the table for Designs 6-10.
- Think about the relationship between the width of the pool, the perimeter of the pool and the size of the deck... Use this relationship to find and record the size of the deck for a pool with a width of 30 feet.
- Using this same relation, write an expression to represent the size of the deck for a pool with width “w.”

Design #	Width of Pool (ft)	Perimeter of Pool (ft)	Size of Deck (ft ²)
1	1	4	8
2	2	8	12
3	3	12	16
4	4	16	20
5	5	20	24
6	6	24	28
7	7	28	32
8	8	32	36
9	9	36	40
10	10	40	44
30	30	120	124
w	w	4w	4w + 4

Part D: Written Reflections to Apply, Connect, and Extend Knowledge

Answer each of the following questions by typing your response into the text box below the question. Please choose a new font (type, style, and/or color) for your responses. All answers must be written in complete and grammatically correct sentences.

1. Explain how you constructed the sketches for Pool Designs 3-5.

Responses will vary.

2. Refer to “Table 1: Pool Area” on page 3. Describe any patterns you notice.

Responses will vary. Students may notice that the area is equal to the width of the pool squared (explicit pattern). They may also notice that the areas are all perfect squares. Students may also notice the change in the area as the width increases by 1 (recursive pattern).

3. Explain how you can find the area of a pool when you are given either the design number or the width of the pool. Give an example to support your response.

Response should include squaring the width or multiplying it by itself. Response may also include that the design number corresponds to the width.

4. What is the width of a pool that has an area of 400 square feet? Explain (in words) how you found your answer.

Correct response: 20 feet.

Explanation should either state that $20 \times 20 = 400$ or that $\sqrt{400} = 20$.

5. Refer to the table you created (on page 4) for finding the size of the deck. Describe any patterns you notice.

Responses will vary.

Students may notice that the deck size increases by 4 ft^2 every time the width increases by 1 foot. They may also notice that the perimeter is always 4 times the width and the deck size is 4 more than the perimeter.

6. What is the relationship between the number of tiles needed to create the deck and the perimeter of the pool?

The deck size is 4 more than the perimeter (represented by the 4 corners).

7. Explain how you can find the size of the deck when you are given either the design number or the width of the pool. Give an example to support your response.

Response should include multiplying the width by 4 and adding 4.

8. What is the area of the pool that has a deck of 52 square feet? Explain how you found your answer.

Correct response: 144 ft^2

$$52 - 4 = 48 \text{ (perimeter of pool)}$$

$$48 / 4 = 12 \text{ (width of pool)}$$

$$12 * 12 = 144 \text{ (area of pool)}$$

For the following questions, assume the cost of building the pool and deck are as follows:

Pool: \$150 per square foot

Deck: \$75 per square foot

9. What is the total cost of Pool Design #8? Show your work or explain how you found your answer.

Correct response: \$12,300

Area of pool = $64 \cdot 150 = \$9,600$

Deck = $36 \cdot 75 = \$2,700$

$9,600 + 2,700 = \$12,300$

10. What is the total cost of Pool Design #20? Show your work or explain how you found your answer.

Correct response: \$66,300

Pool = $20 \cdot 20 = 400 \cdot 150 = \$60,000$

Deck = $4 \cdot 20 + 4 = 84 \cdot 75 = \$6,300$



BONUS: If you have a budget of \$30,000, what is the width of the largest pool you can build? Show your work or explain how you found your answer.

Correct response: 13 feet

Pool: $13 \cdot 13 \cdot 150 = \$25,350$

Deck: $(13 \cdot 4 + 4) \cdot 75 = \$4,200$

Total Cost: $\$25,350 + \$4,200 = \$29,550$